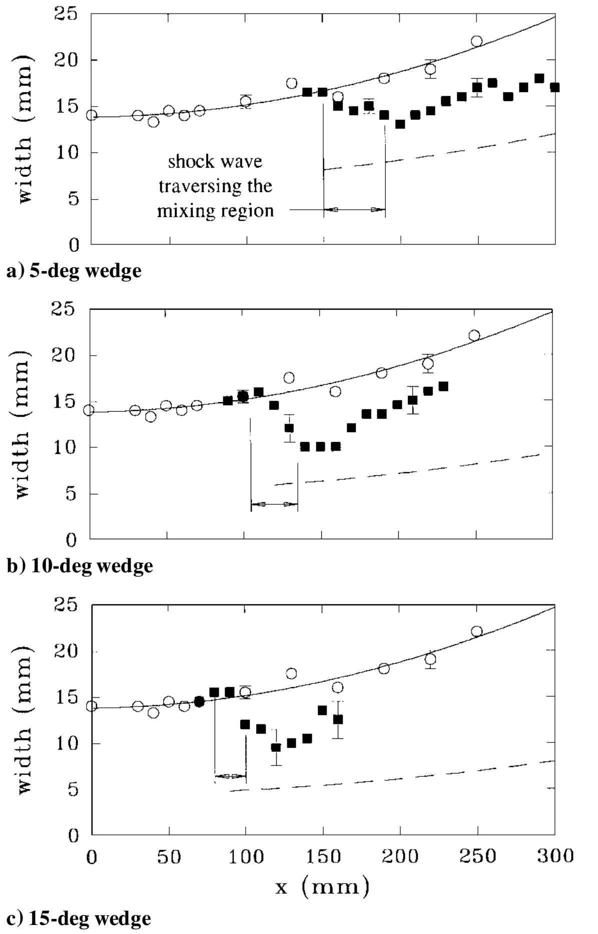

Fig. 5 Measurements and predictions of the mixing region widths from the case 4 schlieren images: o, no shock impingement; □, shock impingement; ——, curve fit for no-shock-impingement experimental data; and – – –, theoretical prediction for postshock region based on inviscid compression model.

Conclusion

Schlieren measurements indicate that the interaction of the oblique shock waves and planar mixing regions in the present study is an unsteady, nonplanar event. No significant differences in the interaction processes were revealed by the schlieren visualization of the four mixing cases. Static pressure measurements on the shock-inducing wedge were predicted reasonably well with the steady, planar interaction analysis. The steady shock trajectory analysis was also in reasonable agreement with the schlieren measurements; however, there was a tendency to underpredict the shock displacement associated with the shock wave's traverse of the mixing region. The increased shock displacement relative to the theoretical predictions is consistent with the appearance of postshock mixing regions that are significantly wider than the inviscid predictions. These effects might be related to additional mass entrainment into the mixing region; however, additional quantitative measurements are needed to examine this possibility and clarify the role of unsteady and nonplanar effects.

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Monte Carlo Analysis of the Hysteresis Phenomenon in Steady Shock Wave Reflections

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Introduction

THE shock wave reflection phenomenon was first studied by Ernst Mach. More than a hundred years ago, he recorded experimentally two different shock wave reflection configurations, a regular reflection (RR) and a Mach reflection (MR). Intensive research of the reflection phenomenon of shock waves was reinitiated in the early 1940s by von Neumann. Since then, it has been realized that the MR wave configuration can be further divided into more specific wave structures.[1]

The analytical models for describing the RR and the MR wave configurations were initiated by von Neumann. They are known as the two- and three-shock theories, respectively. Both theories make use of the conservation equations across the oblique shock waves together with appropriate boundary conditions.

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Table 1 Values of the transition angles of the hysteresis loop for M_0 = 4.96

	Theoretical,[5] deg	Experimental,[5] deg	Numerical,[7] deg
α(MR → RR)	30.9	30.9	36.5[a]
α(RR → MR)	39.3	37.2	39.7

[a]There is a misprint in the text of Ivanov et al.[7] where α(MR → RR) = 35.5 deg is quoted.

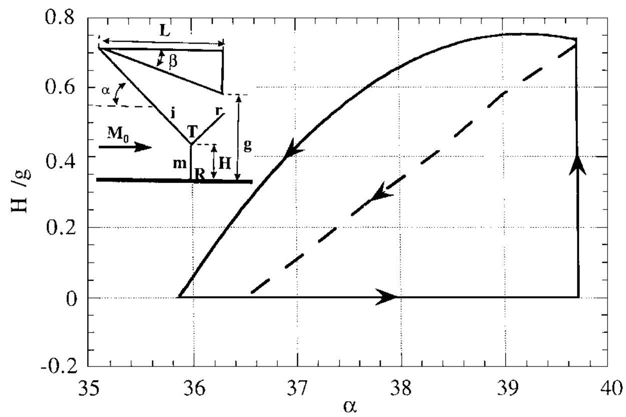


Fig. 1 Normalized Mach stem height as a function of the incident angle for Kn = 0.0057; ——, present procedure with a changing gap and – – –, Ivanov et al.'s[7] procedure with a fixed gap.

Two extreme angles—α_N, the von Neumann angle, and α_D, the detachment angle[1]—at which the RR ↔ MR transition can occur are known. Theoretically, RR is not possible for $\alpha > \alpha_D$ and MR is not possible for $\alpha < \alpha_N$. In the range $\alpha_N \leq \alpha \leq \alpha_D$, which is known as the dual solution domain, both RR and MR are theoretically possible.

The existence of the dual solution domain led, in the early 1980s, to the hypothesis that a hysteresis can exist in the RR ↔ MR transition.[2] A subsequent experimental study failed to record the hysteresis phenomenon.[3] The fact that no experimental evidence of this hypothesis was reported was attributed to the belief that the RR is unstable in the dual solution domain.

Based on the principle of minimum entropy production, it was shown analytically,[4] in the mid 1990s, that the RR is stable in the dual solution domain. Soon after, the hysteresis phenomenon in the RR ↔ MR transition was recorded experimentally for the first time.[5] Then the existence of both RR and MR in the dual solution domain [for the same flow Mach number M_0 and reflecting wedge angle β but different values of g (see the insert in Fig. 1)] was demonstrated numerically[6] using an FCT (flux-corrected transport)-based algorithm. Following these studies, numerical simulations based on the direct simulation Monte Carlo (DSMC) method[7,8] and the total variation diminishing algorithm[8,9] demonstrated the hysteresis phenomenon for the first time. The theoretical, experimental, and numerical RR → MR and MR → RR transition angles for $M_0 = 4.96$ as obtained in the studies are summarized in Table 1. It is evident from Table 1 that, whereas the experimental value[5] of α(MR → RR) is identical to the theoretical one, the experimental value of α(RR → MR) is more than 2 deg smaller than the theoretical one. This could be attributed to three-dimensional effects in the experiment. An opposite situation is obtained when the numerical values[7] are considered. Now the numerical value of α(RR → MR) is very close to the theoretical one and the numerical value[7] of α(MR → RR) is 5.6 deg larger than the theoretical one.

Present Study

The DSMC-based numerical method used by Ivanov et al.[7] enabled one to account for the gas viscosity and the finite thickness of the shock waves. Unfortunately, however, the DSMC computations of flows with Knudsen numbers $Kn \leq 0.005$ that are fairly close to the continuum regime ($Kn = \lambda/L$, where λ is the mean free path in the freestream and L is the length of the wedge; see insert in Fig. 1) are time consuming.

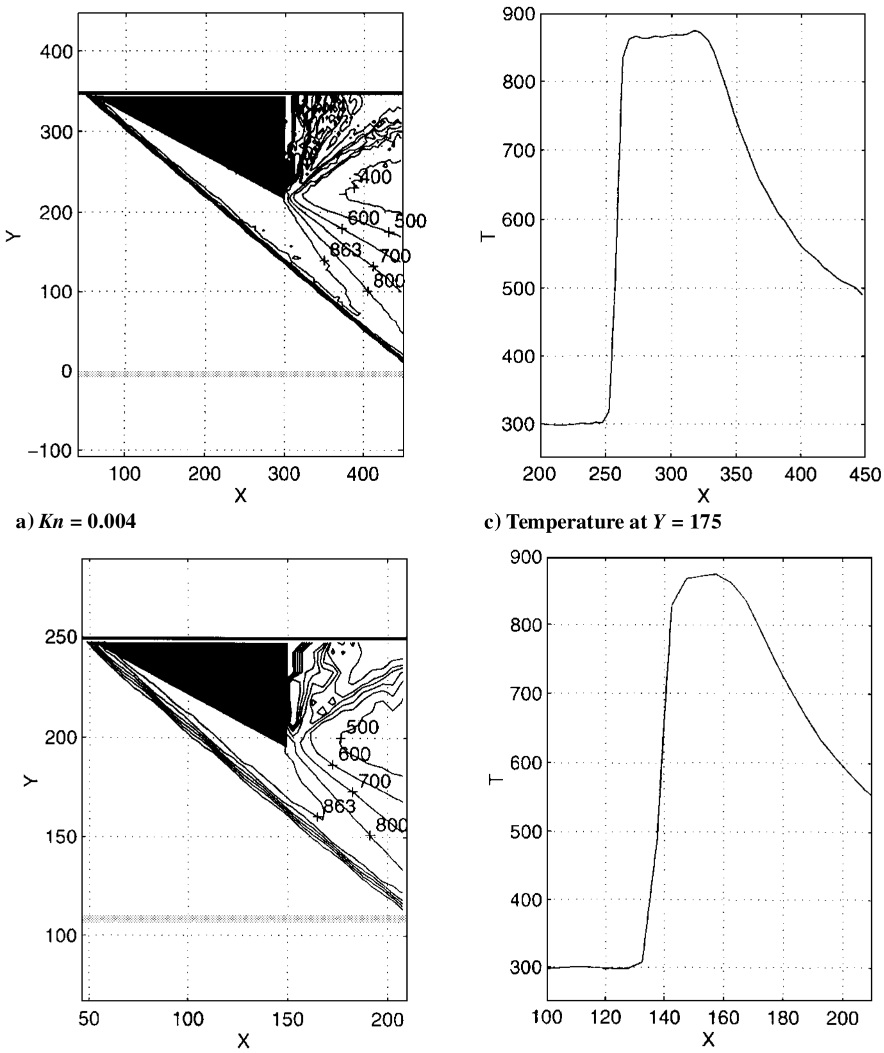
Although in their calculations an adaptive grid was used, in our simulations a rectangular domain and rectangular grid with a constant cell size (about one mean free path) were used. The hard sphere model was adopted for molecular collisions simulation where the energy exchange between the translational and the rotational modes was described by the Larsen–Borgnakke model.[10] Note that the results [α(RR → MR), α(MR → RR)] obtained with the variable hard sphere model[10] and those obtained using the hard sphere model are practically the same. Owing to symmetry only one-half of the computational domain was considered and specular reflection was assumed at the symmetry plane. Freestream conditions with a Maxwellian distribution function were assigned at the upstream boundary. Because the oncoming flow was supersonic, vacuum boundary conditions could be used as the downstream boundary conditions whereby molecules that left the computational domain through the downstream boundary were removed from the computation. Unfortunately, however, these (upstream and downstream) conditions introduced a systematic error because of the counterflow velocity component, which depended on the freestream velocity.[10] For a flow with Mach number $M_0 = 5$, the error of the freestream velocity was less than 2%. The problem was surmounted by using a moving piston at the upstream and downstream boundaries whereby the molecules were reflected specularly from the wall of the piston that moved with the freestream velocity. In the calculations, nitrogen ($\gamma = 1.4$) with $M_0 = 4.96$ and $T_0 = 300$ K was used as the freestream gas. About 700,000 molecules were used in the simulations. Specular reflection conditions were taken as the boundary conditions at the wedge surface to avoid the boundary-layer formation, which is known to stipulate a change in the shock wave angle of incidence.

The calculations showed that the angles of incidence of the wedge-generated shock waves were close to their corresponding theoretical values (the discrepancy was less than 3%). This small discrepancy is quite understandable owing to an inherent error arising from the fact that the shock wave has a finite thickness of about 10–15 mean free paths. The flow parameters calculated behind the incident shock wave differed from their respective theoretical values by no more than 1%.

The first series of computations was carried out for a set of fixed wedge angles generating incident shock waves with a specified angle of incidence α. A freestream gas flow into which a wedge was instantly inserted at $t = 0$ was used as an initial condition. A comparison between the results of these calculations for $Kn = 0.004$ and 0.01 is shown in Figs. 2a and 2b, respectively. The temperature profiles at $Y = 175$ for these two cases are shown in Figs. 2c and 2d, respectively. It is clearly seen that the flow behind the incident shock wave is not uniform for the $Kn = 0.01$ case. Furthermore, for this case the size of the region between the incident shock wave and the leading characteristic of the expansion fan emanating from the trailing edge of the reflecting wedge is only about two shock wave thicknesses. It is much larger for the $Kn = 0.004$ case (Fig. 2a), for which a uniform flow region is clearly seen (Fig. 2c).

Rotating wedge simulations were performed in the following way. At the first step a solution for the case $\alpha_i > \alpha_D$, for which only an MR is theoretically possible, was obtained. Then the computation for $\alpha_{i+1} < \alpha_i$ was performed in such a way that the solution for the case with α_i was used as the initial condition for the solving case with α_{i+1}. This procedure was repeated until a case with $\alpha_f < \alpha_N$, for which only an RR is theoretically possible, was solved. Then the direction of the rotation of the reflecting wedge was reversed, and α was increased until the initial value of α_i was reached. The preceding procedure ensured that during the rotations from α_i to α_f and back from α_f to α_i the MR → RR and the RR → MR transitions were encountered, respectively.

The preceding calculations were performed in two different ways. In the first, as was done by Ivanov et al.,[7] the reflecting wedge was rotated around its trailing edge, and the gap g between the trailing edge and the symmetry plane (see insert in Fig. 1) was kept constant. In the second, the gap was continuously adjusted in such a way that the reflected shock wave passed close to the trailing edge of the reflecting wedge but did not hit it. This procedure enabled us to reduce the influence of the expansion wave at large Knudsen



a) *Kn* = 0.004
c) Temperature at *Y* = 175
b) *Kn* = 0.01
d) Temperature at *Y* = 175

Fig. 2 Oblique shock waves over a reflecting wedge for M_0 = 4.96 and β = 28 deg.

numbers and therefore to reduce the influence of kinetic effects on the MR $\rightarrow$ RR transition.

Figure 1 shows the hysteresis loops as calculated using Ivanov et al.[7] (constant gap and an adaptive grid) and our (adjustable gap and a fixed grid) procedures. The procedure of Ivanov et al.[7] (dashed line) results in α(RR $\rightarrow$ MR) = 39.7 deg and α(MR $\rightarrow$ RR) = 36.5 deg in agreement with the value given in their Fig. 5a but not in the text of their paper. The value of these angles as calculated using our procedure (solid line) results in α(RR $\rightarrow$ MR) = 39.7 deg and α(MR $\rightarrow$ RR) = 35.8 deg. Apparently, the difference between the experimental α(MR $\rightarrow$ RR) and that obtained by the Monte Carlo simulations is caused by kinetic effects. Indeed, as can be seen from Fig. 2, the width of the shock wave for $Kn = 0.01$ is quite large, and the shock wave is even smeared farther due to the interaction with the expansion fan. Both of these effects do not allow us to resolve the moment of the MR $\rightarrow$ RR transition with a reasonable accuracy for *Kn* values larger than those used both in the present calculations and in those of Ivanov et al.[7]

Conclusions

The hysteresis phenomenon in the regular-Mach reflection transition in steady flow has been reconsidered using modified and simplified DSMC methods. The discrepancy between the experimental and the numerical values of the α(MR $\rightarrow$ RR) as obtained using the Monte Carlo simulations was explained by kinetic effects.

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